Non-intrusive adaptive surrogate modeling of parametric frequency-response problems

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TARGET PROBLEM

Parametric frequency-domain LTI system of size $\gg 1$:

$$\boxed{\omega \in \mathbb{C},\ \theta \in \mathbb{R}^d \quad \rightsquigarrow \quad \text{find } \mathsf{x}(\omega,\theta)\ :\ \omega\mathsf{E}(\theta)\mathsf{x}(\omega,\theta) = \mathsf{A}(\theta)\mathsf{x}(\omega,\theta) + \mathsf{b}(\omega,\theta)}$$

Objectives:

- **Non-parametric MOR**: approximate $\omega \mapsto \mathsf{x}(\omega,\theta)$ for a fixed θ. This is often done with *rational functions*, i.e., ratios of polynomials.
- **Parametric MOR**: approximate $(\omega,\theta) \mapsto \mathsf{x}(\omega,\theta)$. This is much more complicated, since poles and residues (wrt ω) will depend on θ.

STRUCTURE OF SURROGATE MODEL

We seek an approximation in **parametric rational pole-residue format**:

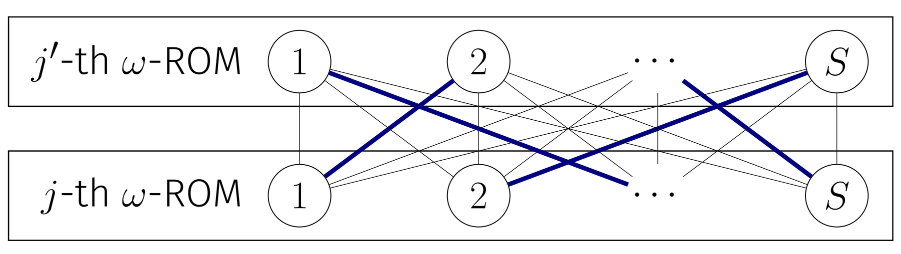
$$\boxed{\mathsf{x}(\omega,\theta) \approx \sum_i \frac{\mathsf{r}_i(\theta)}{\omega - \lambda_i(\theta)} = \sum_i \frac{\sum_j \mathsf{r}_{ij}\phi_j(\theta)}{\omega - \sum_j \lambda_{ij}\phi_j(\theta)}}$$

The functions ϕ_j are a given scalar-valued basis, e.g., hat functions, polynomials, RBFs, ..., so we only need to find the expansion coefficients $\{\mathsf{r}_{ij}\}_{ij}$ and $\{\lambda_{ij}\}_{ij}$. To this aim, we first gather **data: univariate ω-ROMs in pole-residue form**

$$\boxed{\text{given } \{\theta_j\}_j \subset \mathbb{R}^d\text{, build ROMs } \left\{\widetilde{\mathsf{x}}_j(\omega) = \sum_i \frac{\mathsf{r}_i^{(j)}}{\omega - \lambda_i^{(j)}} \approx \mathsf{x}(\omega,\theta_j)\right\}_j}$$

MATCHING STRATEGY

Before we can use $\{\lambda_i^{(j)}\}_{ij}$ to find $\{\lambda_{ij}\}_{ij}$ (and the same for the r's) we need to match the poles for different j's, otherwise **we might be combining information pertaining to different resonating modes!** For this, we look for *optimal permutations* of the terms in the pole-residue expansions: we build a weighted graph

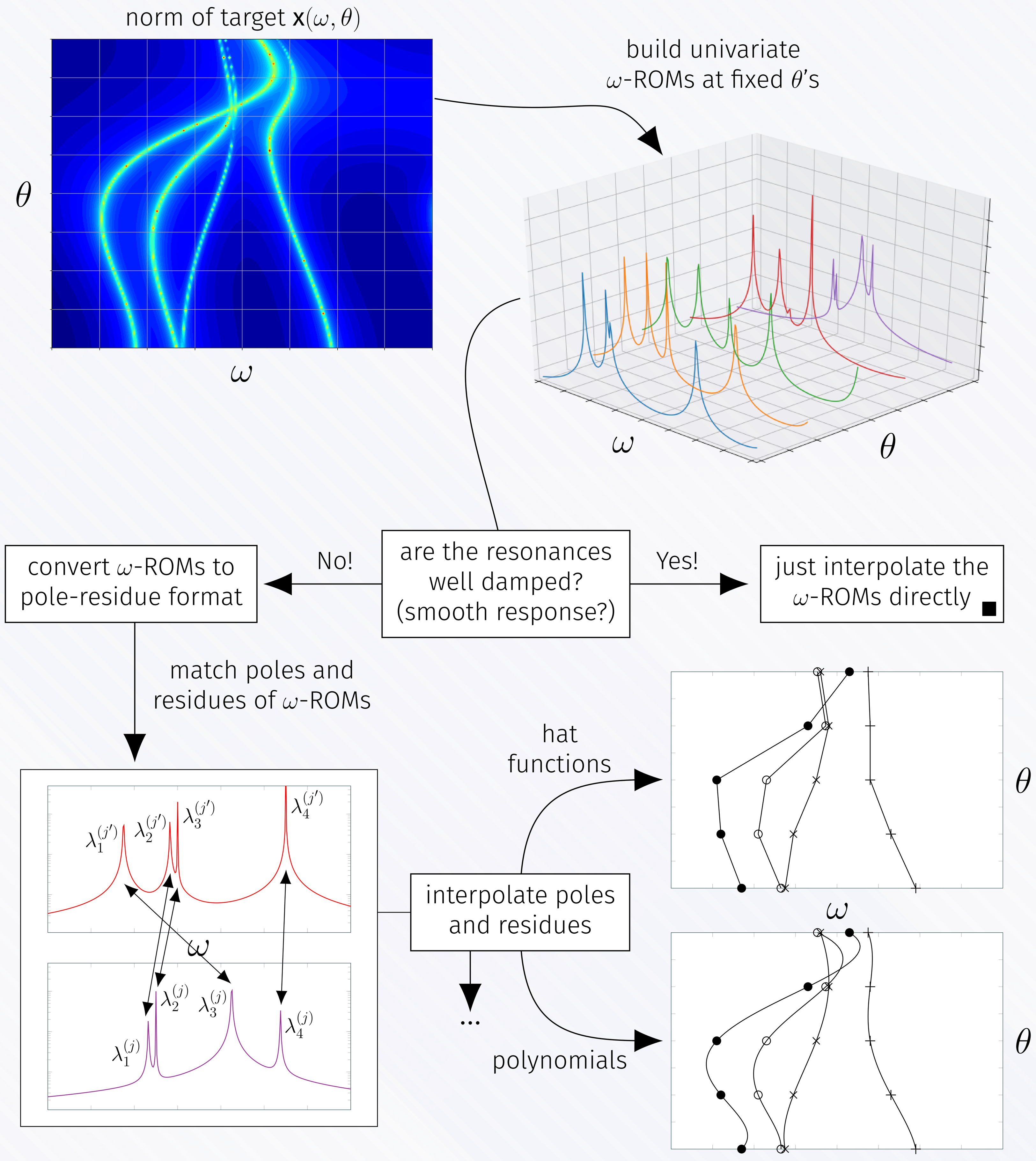


with weights

$$w_{ii'}^{(jj')} = \left|\lambda_i^{(j)} - \lambda_{i'}^{(j')}\right| + \gamma\left\|\mathsf{r}_i^{(j)} - \mathsf{r}_{i'}^{(j')}\right\|$$

and we find a "matching" set of edges that **minimizes the total sum of weights**.

SAMPLE WORKFLOW

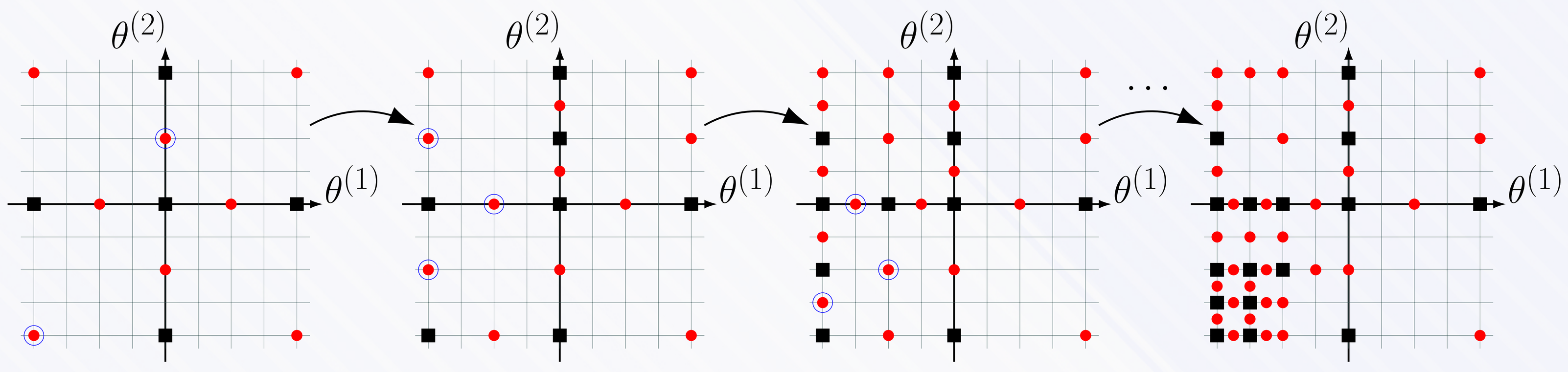


NON-INTRUSIVE ADAPTIVE SAMPLING VIA LOCALLY ADAPTED SPARSE GRIDS

Sparse grids allow for structured localized refinements in arbitrary θ-dimension d: given a set of SG points, we can define a discrete hierarchical SG neighborhood.
We can use this for non-intrusive adaptive sampling over θ:

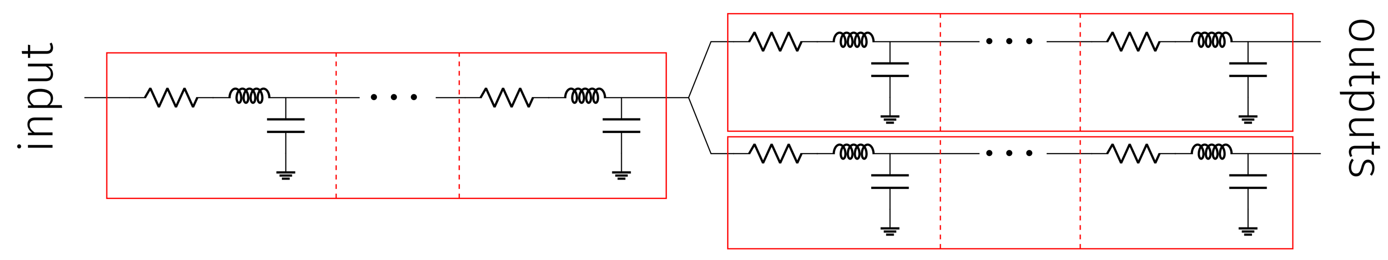
- We use a collection of SG points (**not necessarily an SG in the usual sense**) as training set. These are the black squares.
- We use their discrete neighborhood as test set. These are the red dots. **If a point of the test set is badly approximated, we move it to the training set** (blue circles).

This requires expensive computations at all points of the training and test sets.



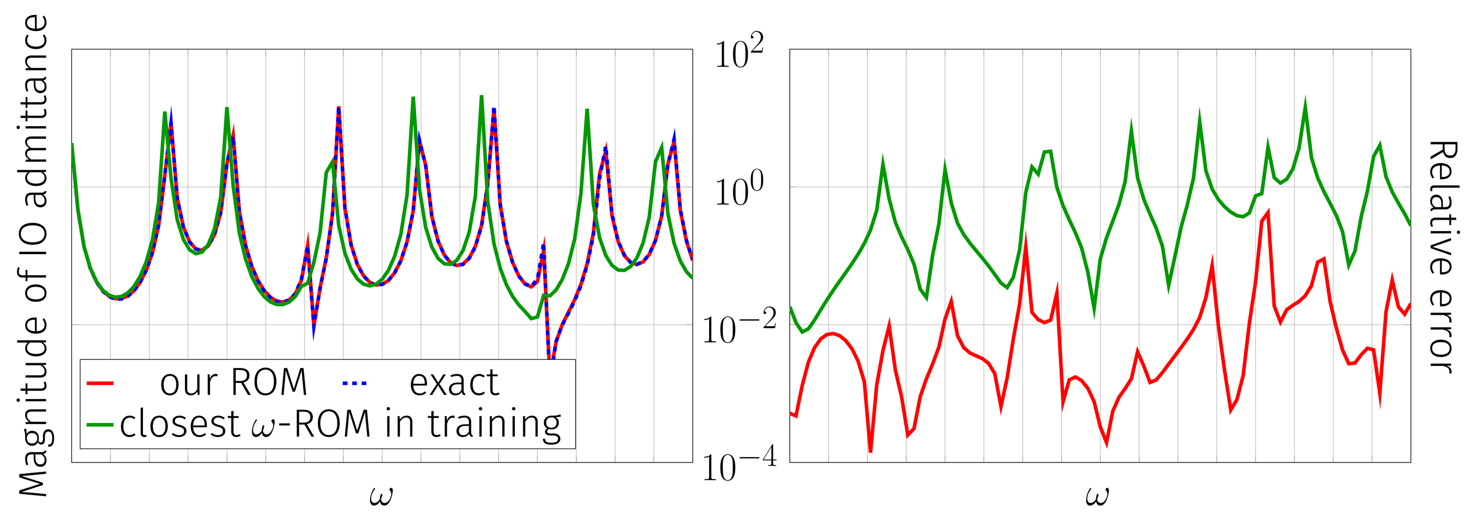
NUMERICAL RESULTS: IMPEDANCE OF PARAMETRIC TRANSMISSION LINE

Problem: Y-transmission line composed of RLC blocks

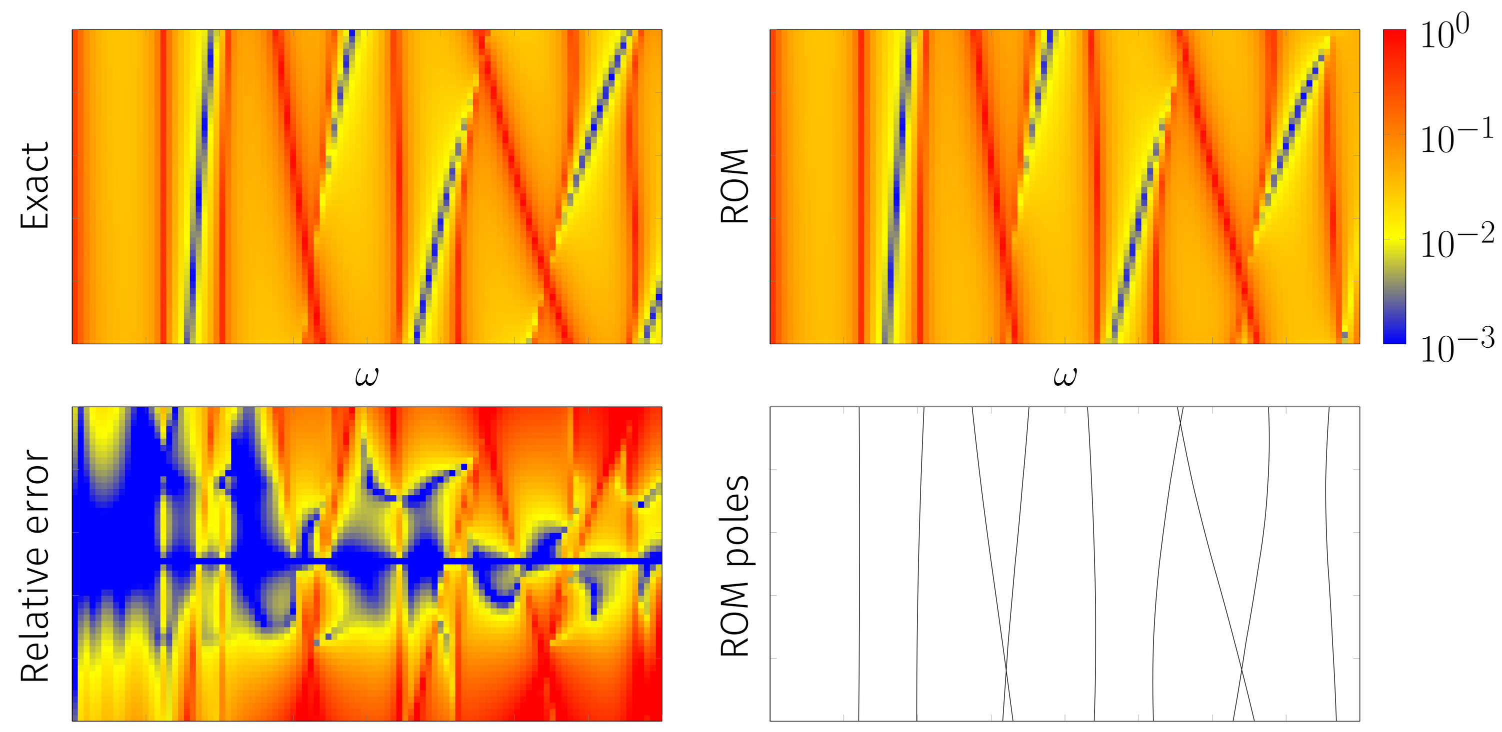


$(\omega,\theta) \in \mathbb{C}\times\mathbb{R}^9$ denote **frequency** and **average R/L/C values** (in each branch)

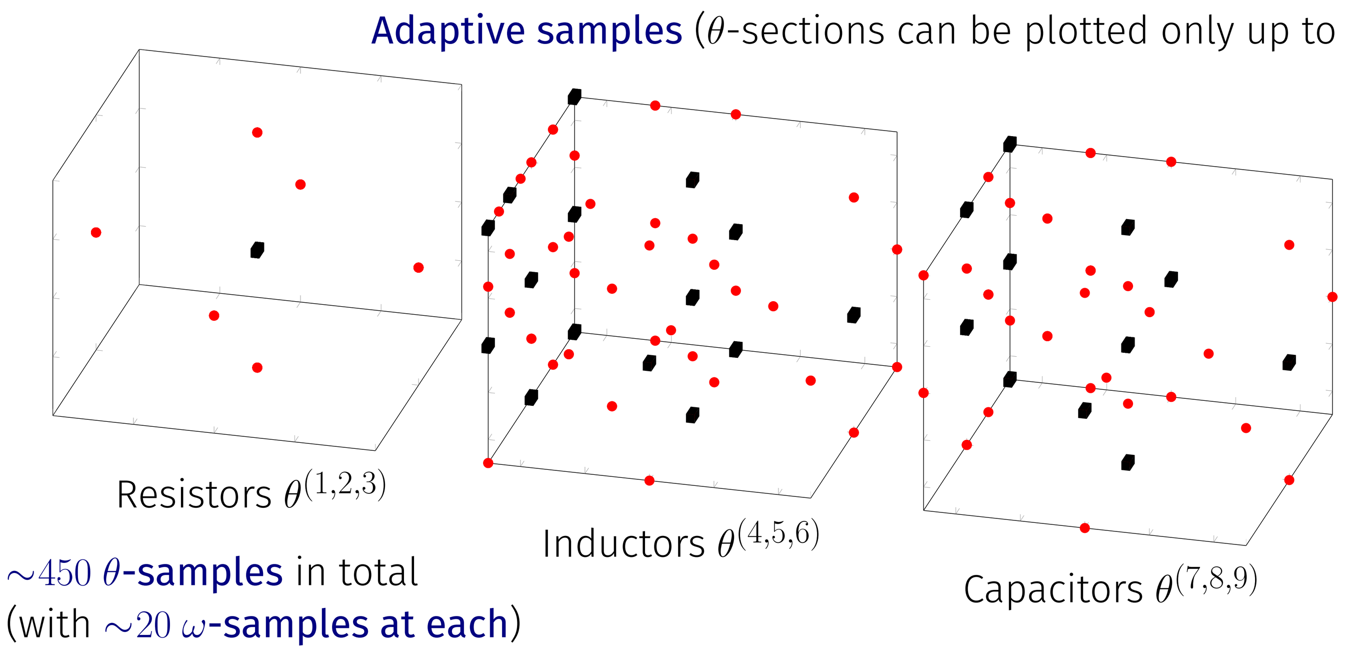
Validation at a (random) not-yet-seen value of θ



Validation along a diagonal "section" of θ-space (y-axis sweeps the diagonal)



Adaptive samples (θ-sections can be plotted only up to 3D)



~ 450 θ**-samples** in total
(with ~ 20 ω-samples at each)

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